

# Overview of Several Subcritical Measurement Methods

J. Hutchinson, S. Melton, B. Rooney,  
W. Myers, A. Sood

Los Alamos National Laboratory  
N-2 (Advanced Nuclear Technology) and  
XCP-7 (Transport Applications)

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# Introduction

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- Time-Correlation Analysis
  - Rossi-Alpha
  - Interval distribution (Babala)
  - Correlation analysis
- Moments Analysis
  - Feynman variance (Hansen-Dowdy or Hage-Cifarelli)
  - Bennett Variance
- Probability of Neutron Detection
  - Zero-Count probability (Mogilner)

# Introduction

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- Most methods were developed for zero power reactors and later applied to subcritical systems or power reactors.
- Most of the work took place from 1940s-1960s.
- Differ in energy response, timing required, detection system required, multiplication over which the method is valid, etc...
- All data shown were taken on the bare BeRP ball using the NPOD detector system.

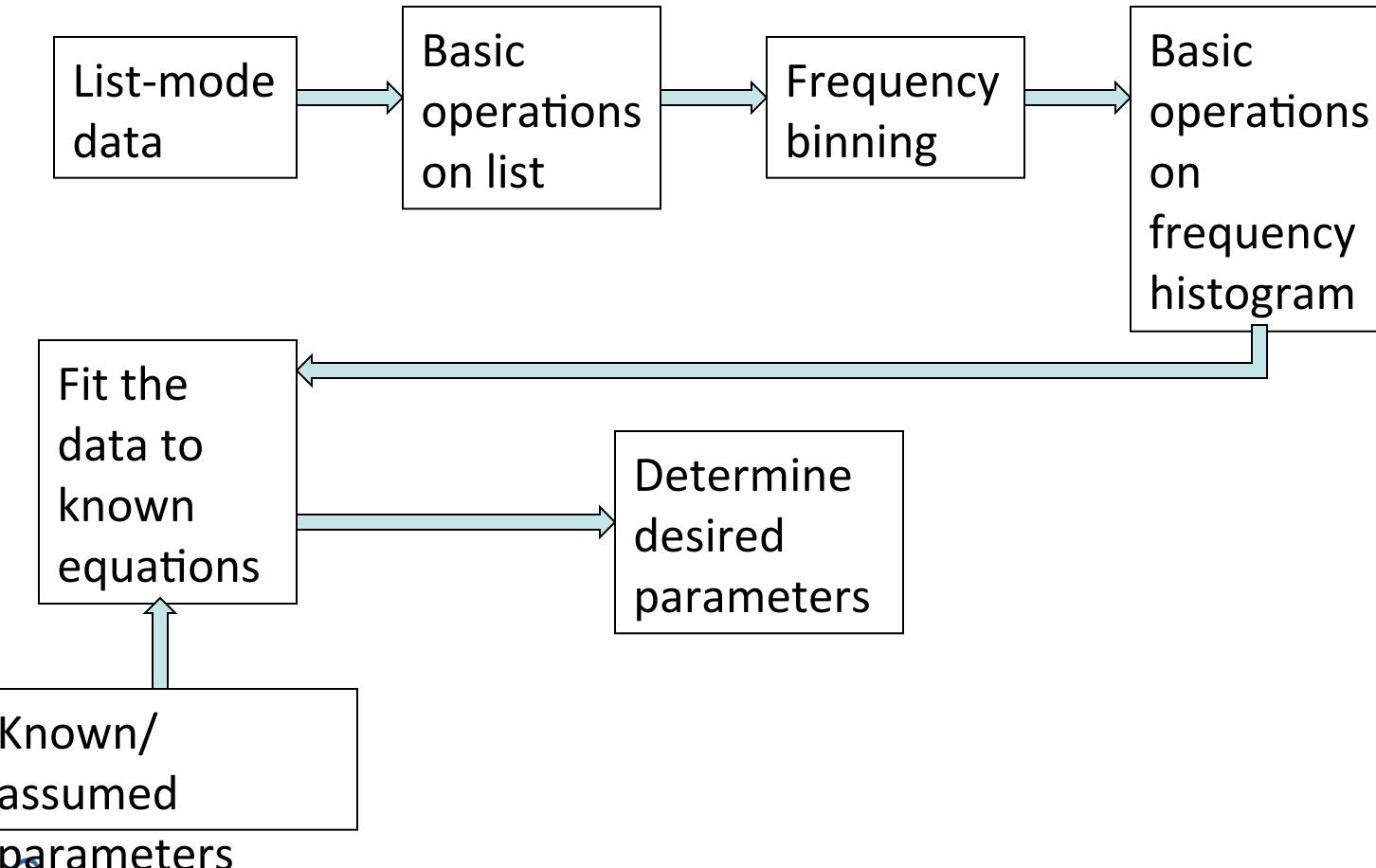
# Diven's parameter

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- Diven's parameter is different from "D number"
- ~0.82 for Pu-240 spontaneous fission

$$D_v = \frac{\bar{v}^2 - \bar{v}}{\bar{v}^2}$$

# General order of operations for time-domain analysis



# Rossi-alpha

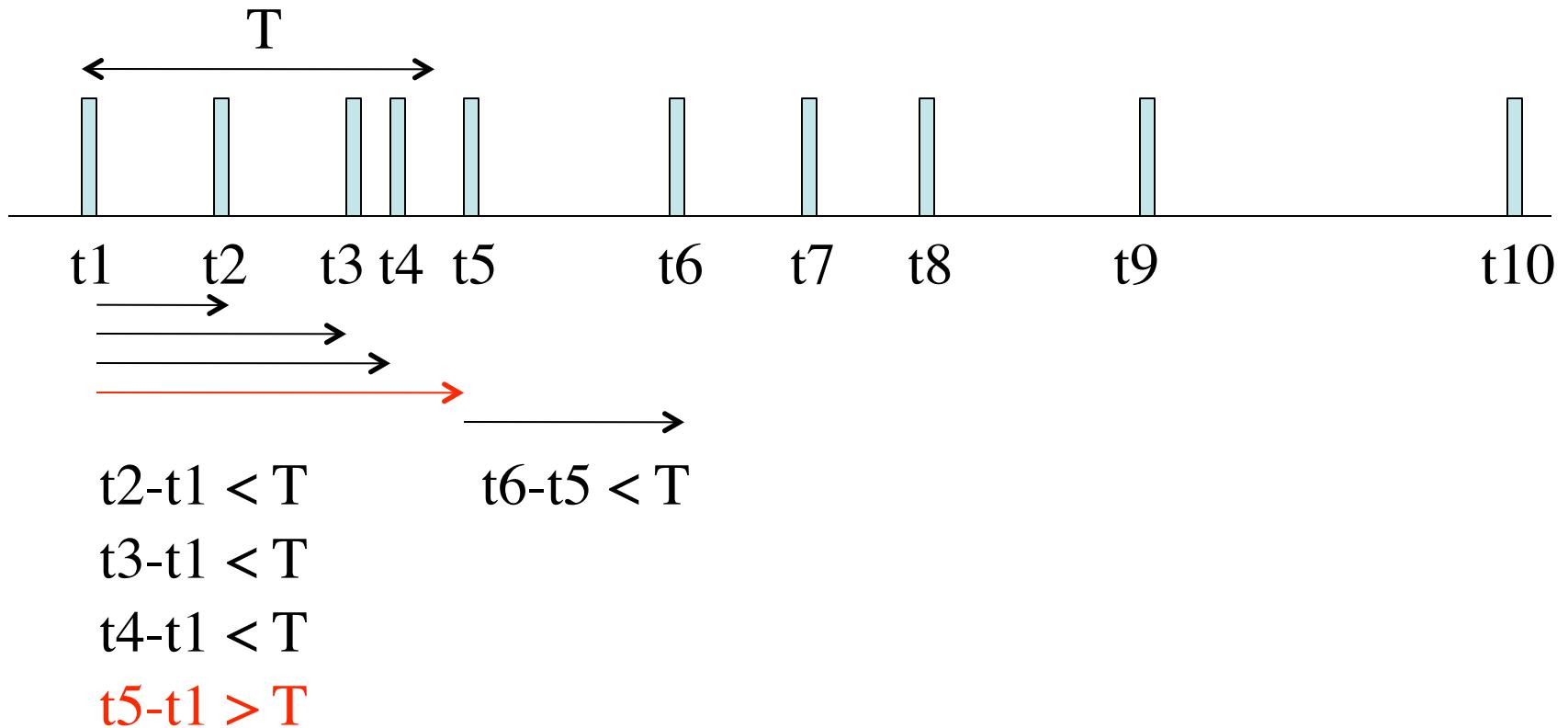
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- Feynman, de Hoffman, and Serber in 1956.

$$p(t)\Delta = A\Delta + Be^{-\alpha t}\Delta$$

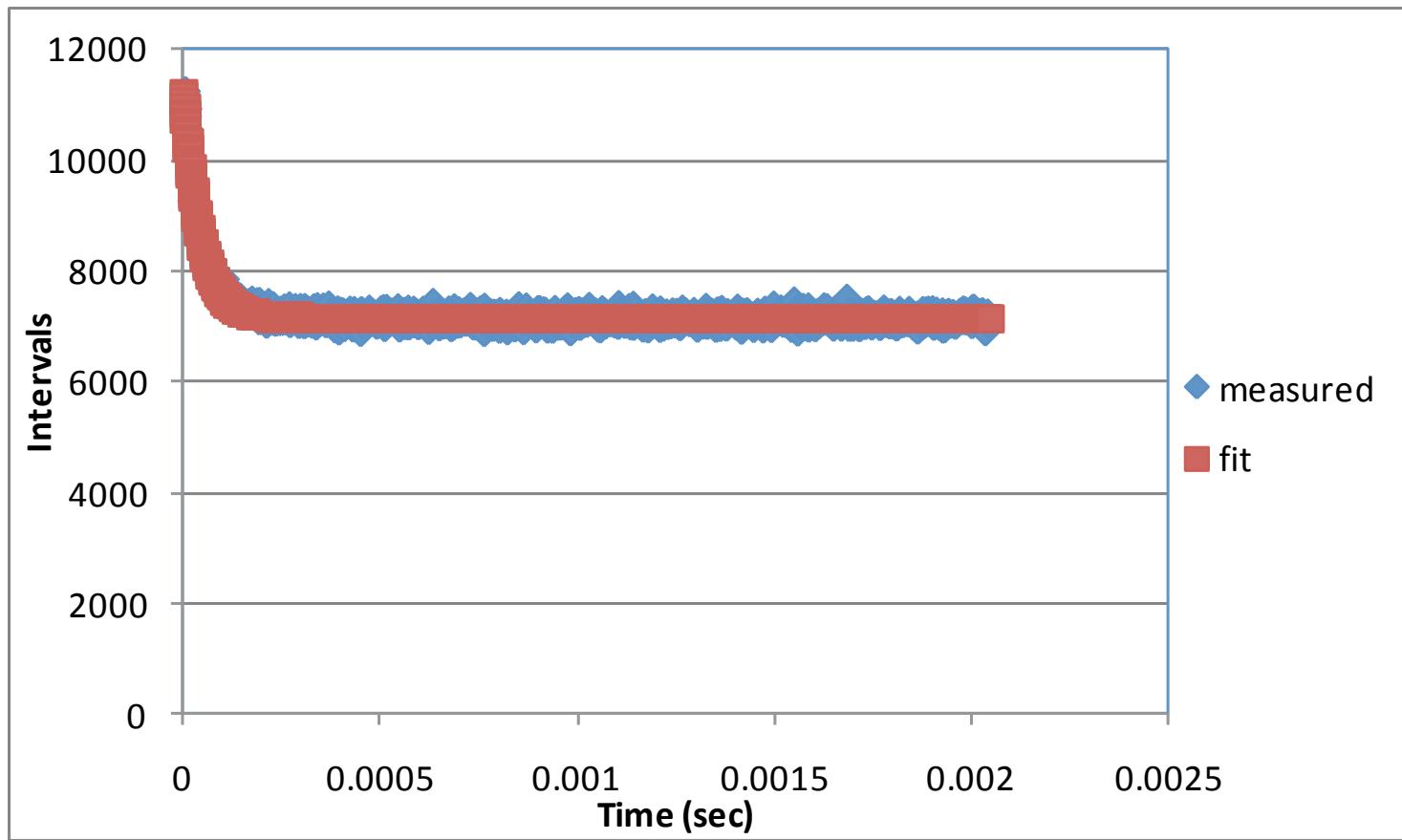
# Rossi-alpha

- Perform frequency binning on “t2-t1”, “t3-t1”, etc...



U N C L A S S I F I E D

# Rossi-alpha



U N C L A S S I F I E D

# Rossi-alpha

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- Fit solves for parameters A, B, and  $\alpha$

$$A = F\epsilon$$

$$\alpha = \frac{\rho_p}{\Lambda} = \frac{1 - k_p}{l}$$

$$B = \frac{\varepsilon D_v k_p^2}{2\alpha l^2}$$

$$\rho_p = \frac{1 - k_p}{k_p}$$

# Rossi-alpha

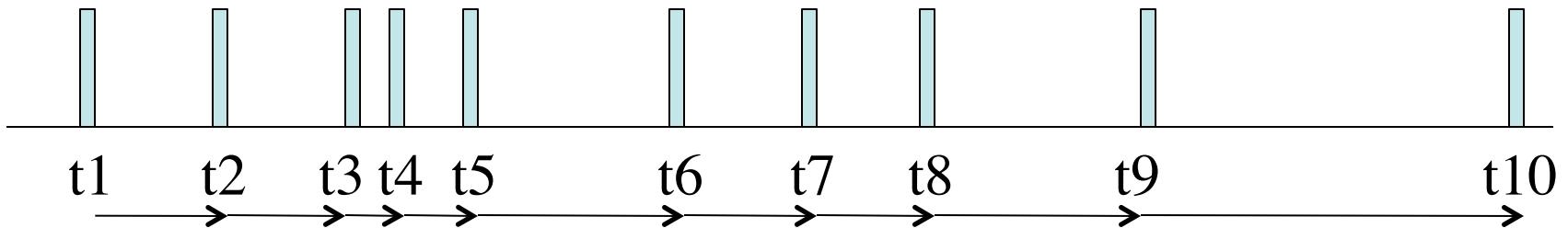
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- Parameters:
  - $F$  is the fission emission rate
  - $\epsilon$  is the efficiency (cts/fission)
  - $\ell$  is the neutron lifetime
  - $\Lambda$  is the neutron generation time
- We use:
  - Assumed/known parameters:  $F$  and  $D_v$
  - Solve for:  $k_p$ ,  $\epsilon$ , and  $\Lambda$  (or  $\ell$ )

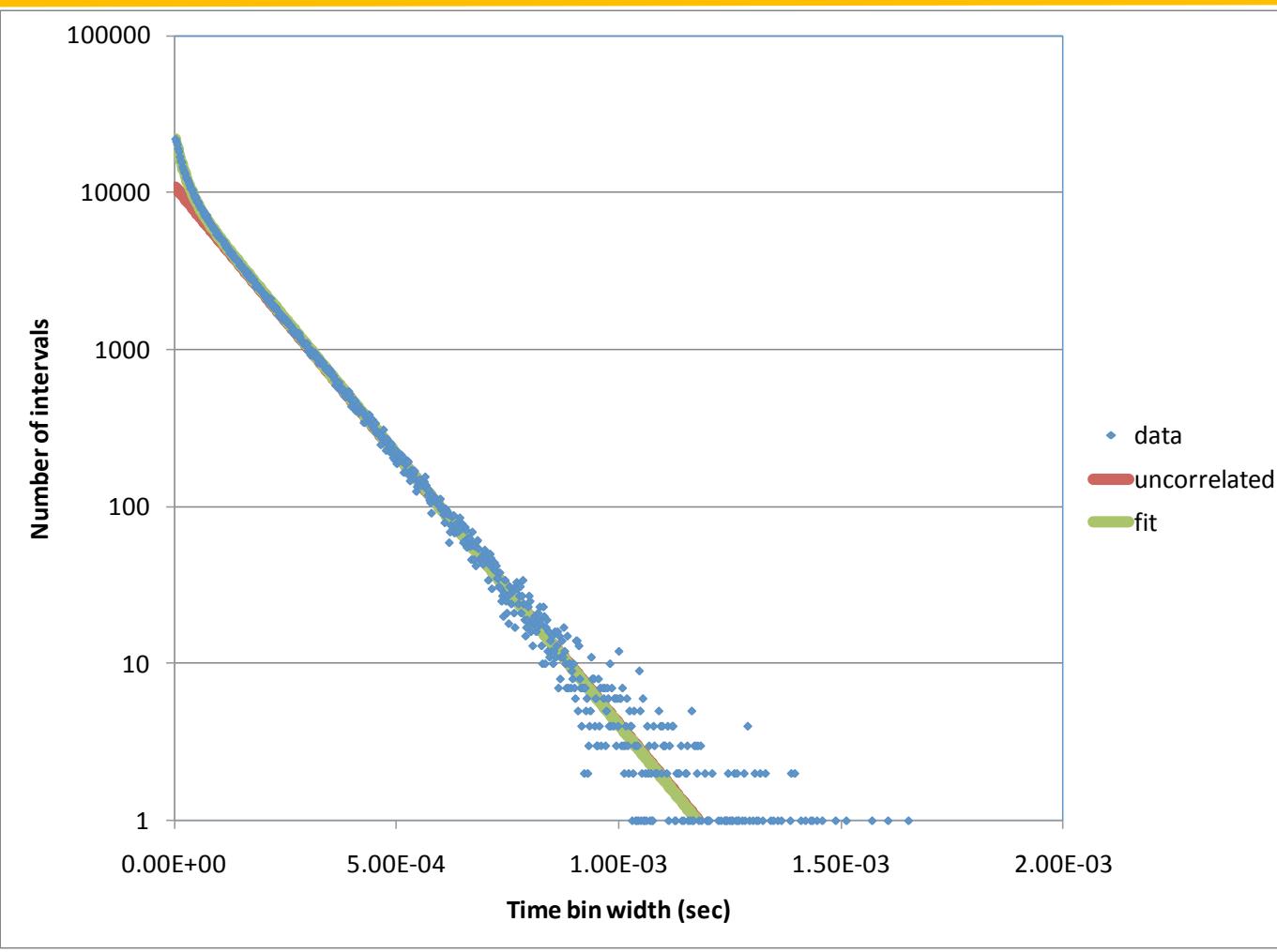
# Interval distribution

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- Babala in 1966
- Perform frequency binning on “t<sub>2</sub>-t<sub>1</sub>”, “t<sub>3</sub>-t<sub>2</sub>”, etc...



# Interval distribution



# Interval distribution

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$$P_{CC}(\tau) = k \exp \left\{ -\sigma \left[ (\gamma - 1)\alpha\tau + 2 \ln \frac{T}{4\gamma} \right] \right\} \left( \frac{2S}{T} \right)^2 \left[ 1 + \frac{2\gamma^2}{\sigma S^2} \exp(-\alpha\gamma\tau) \right]$$

$$S = (\gamma + 1) + (\gamma - 1) \exp(-\alpha\gamma\tau)$$

$$T = (\gamma + 1)^2 + (\gamma - 1)^2 \exp(-\alpha\gamma\tau)$$

- Fit to find  $k$ ,  $\alpha$ ,  $\sigma$ , and  $\gamma$ .

$$\sigma = \frac{sA}{\bar{\nu}D_\nu} \quad \gamma = \sqrt{\left( 1 + \frac{2\varepsilon D_\nu}{\rho_p^2} \right)} \quad C = \frac{\sigma\alpha}{2}(\gamma^2 - 1)$$

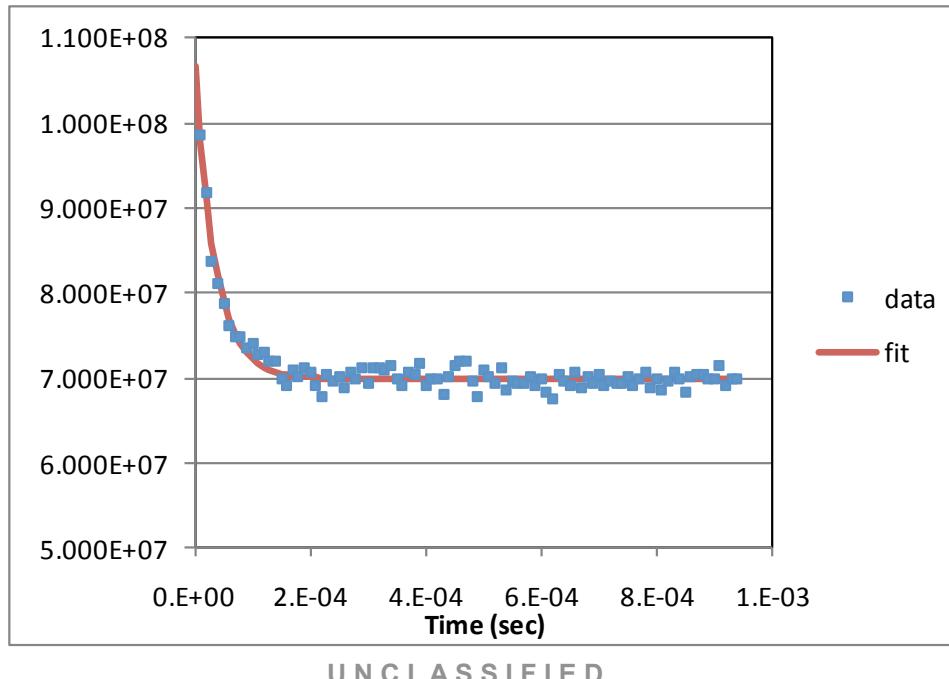
# Interval distribution

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- Parameters
  - $s$  is the starter neutron emission rate
  - $C$  is the detector count rate (cts/sec)
  - Other terms same as previous definitions
- We use:
  - Known parameters:  $s$ ,  $\bar{\nu}$  and  $D_v$
  - Solve for:  $k_p$ ,  $\epsilon$ , and  $\Lambda$ (or  $\ell$ )

# Correlation analysis

$$\phi_{xy}(k\Delta) = \frac{1}{N - k} \sum_{i=1}^{N-k} x[t + i\Delta]y[t + (i + k)\Delta]$$



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# Correlation analysis

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- Constants are the same as in the Rossi-alpha method
- Can solve for the same 3 parameters:  $k_p$ ,  $\epsilon$ , and  $\Lambda$ (or  $\ell$ )

$$\phi_{xy}(\tau) = A^2 + ABe^{-\alpha\tau}$$

$$\phi_{xx}(\tau) = A^2 + ABe^{-\alpha\tau} + A\delta(\tau)$$

# Bennett Variance method

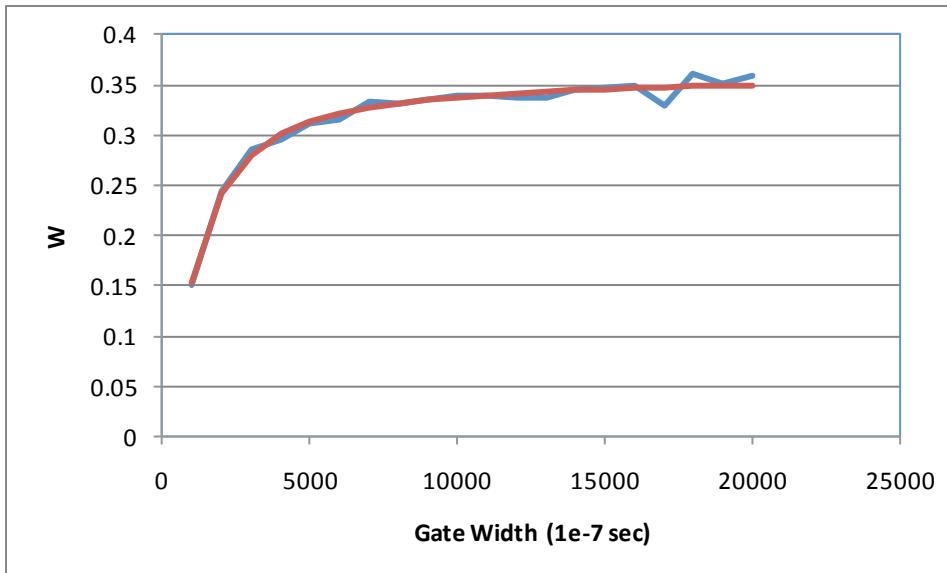
| Time of event | Bins  | Number of events ( $c_k$ ) | $(c_{k+1}-c_k)^2$ |
|---------------|-------|----------------------------|-------------------|
| 2955          | 2000  | 0                          | 9                 |
| 3906          | 4000  | 3                          | 4                 |
| 3954          | 6000  | 1                          | 4                 |
| 5149          | 8000  | 3                          | 1                 |
| 6869          | 10000 | 2                          | 1                 |
| 7185          | 12000 | 1                          | 0                 |
| 7583          | 14000 | 1                          | 4                 |
| 8166          | 16000 | 3                          | 4                 |
| 8816          | 18000 | 1                          | 4                 |
| 10541         | 20000 | 3                          | 9                 |
| 12577         | 22000 | 0                          | 0                 |
| 14699         | 24000 | 0                          | 1                 |
| 15213         | 26000 | 1                          | 1                 |
| 15721         | 28000 | 0                          | 1                 |
| 17610         | 30000 | 1                          | 1                 |
| 18477         |       | 0                          |                   |
| 19228         |       |                            |                   |
| 19744         |       |                            |                   |
| 25913         |       |                            |                   |
| 29017         |       |                            |                   |

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# Bennett Variance method

- Bennett in 1960.

$$W = \frac{\langle (c_{k+1} - c_k)^2 \rangle}{2\langle c_k \rangle} - 1 \quad W = \frac{\varepsilon D_v}{\rho_p^2} \left( 1 - \frac{\frac{3}{2} + \frac{1}{2}e^{-2\alpha T} - 2e^{-\alpha T}}{\alpha T} \right)$$



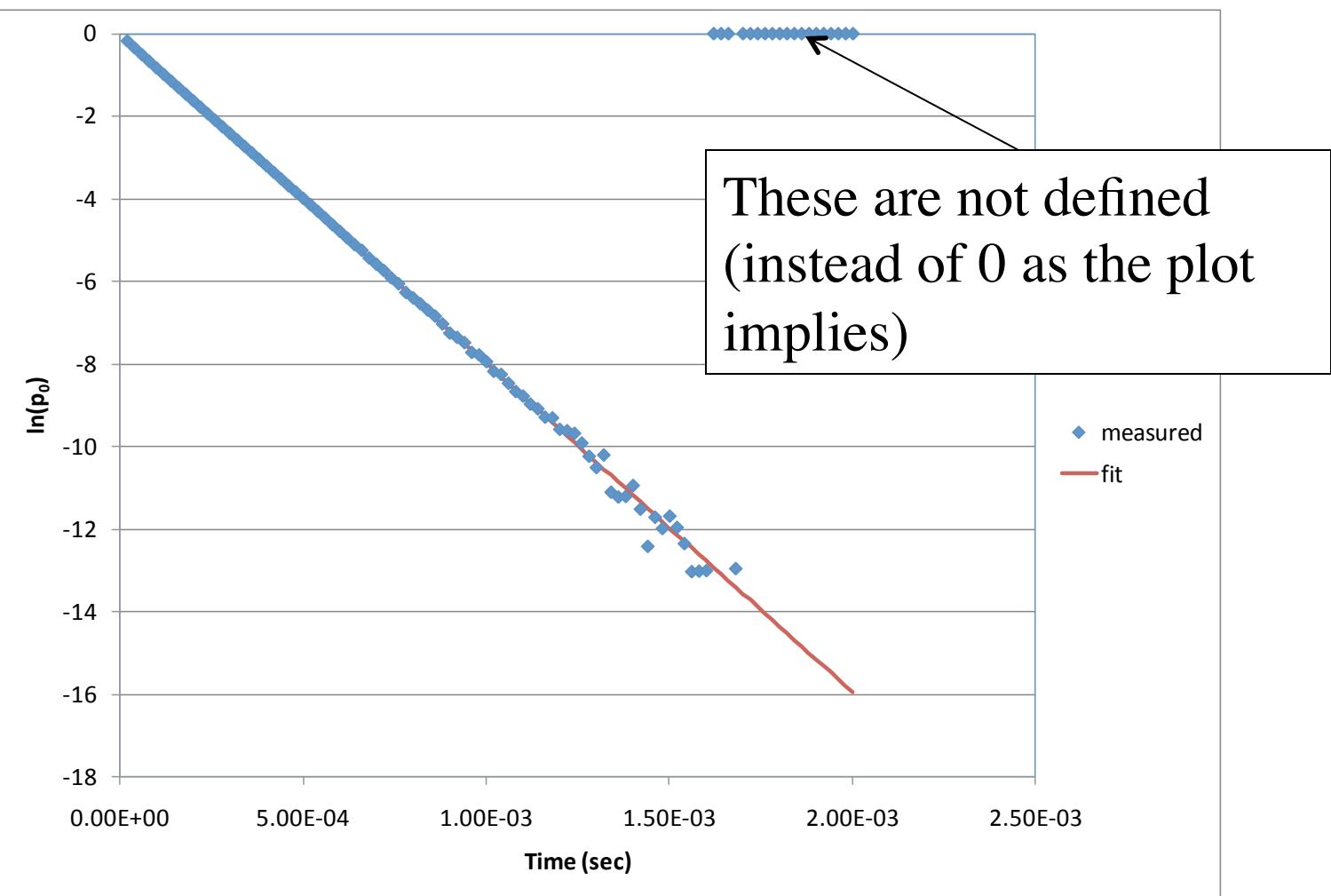
# Zero count probability method

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- Mogilner and Zolotukhin in 1961.
- For these data  $p_0=0.838$  and  $\ln(p_0)=-0.177$

|   | Feynman histogram |          | Probability |          |
|---|-------------------|----------|-------------|----------|
| 0 | N0                | 30236684 | p0          | 8.38E-01 |
| 1 | N1                | 5155232  | p1          | 1.43E-01 |
| 2 | N2                | 608431   | p2          | 1.69E-02 |
| 3 | N3                | 67843    | p3          | 1.88E-03 |
| 4 | N4                | 7826     | p4          | 2.17E-04 |
| 5 | N5                | 955      | p5          | 2.65E-05 |
| 6 | N6                | 119      | p6          | 3.30E-06 |
| 7 | N7                | 19       | p7          | 5.27E-07 |
| 8 | N8                | 4        | p8          | 1.11E-07 |
|   | N                 | 36077113 |             | 1.00E+00 |

# Zero count probability method



# Zero count probability method

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$$\ln p_0(\Delta) = -\frac{2c\Delta}{\gamma + 1} \left\{ 1 + \frac{2}{(\gamma - 1)\alpha\Delta} \ln \left[ \frac{(\gamma + 1)^2 - (\gamma - 1)^2 e^{-\alpha\gamma\Delta}}{4\gamma} \right] \right\}$$

- Fit for  $c$  (count rate),  $\alpha$ , and  $\gamma$ .
- Can use same equations as before to solve for  $k_p$ ,  $\epsilon$ , and  $\Lambda$  (or  $\ell$ )

# Results

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| Method         | kp    | Gen Time (sec) | Lifetime (sec) | Efficiency | Alpha (/sec) | Gamma |
|----------------|-------|----------------|----------------|------------|--------------|-------|
| Hansen-Dowdy   | 0.765 | 9.36E-06       | 7.17E-06       | 5.78E-02   | 2.50E+04     | 1.42  |
| Hage-Cifarelli | 0.790 | 9.36E-06       | 7.17E-06       | 5.78E-02   | 2.50E+04     | 1.53  |
| Rossi          | 0.739 | 1.51E-05       | 1.12E-05       | 5.44E-02   | 2.34E+04     | 1.31  |
| Int dist.      | 0.811 | 7.82E-06       | 6.34E-06       | 2.14E-02   | 2.98E+04     | 1.28  |
| Correlation    | 0.712 | 1.46E-05       | 1.04E-05       | 6.36E-02   | 2.77E+04     | 1.28  |
| Bennett        | 0.741 | -              | -              | -          | 2.22E+04     | -     |
| O prob         | 0.722 | 1.05E-05       | 7.56E-06       | 7.07E-02   | 3.68E+04     | 1.33  |
| KCODE          | 0.774 | -              | -              | -          | -            | -     |
| Count rate     | -     | -              | -              | 6.89E-02   | -            | -     |

# Conclusions/future work

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- Most of the methods worked well for this proof of concept study.
- These methods will be reevaluated for this and other data sets using Sheila's code.
- Additional methods will be added next year.